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AN EXPERIMENTAL STUDY OF STOCHASTIC PHASE SYNCHRONIZATION IN VERTICAL CAVITY LASERS

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The response of a bistable vertical cavity surface emitting laser to a periodic input signal display all the hallmarks of stochastic resonance. Accurate experimental measurements show the occurrence of noise-induced frequency and effective phase locking characterized by a plateau of the mean output switching rate and a minimum of the relative–phase diffusion constant, respectively.

Keywords: Stochastic resonance; phase synchronization; frequency entrainment; vertical cavity lasers; alternating polarization configurations.

1. Introduction

The phenomenon known as stochastic resonance (SR) is a peculiarity in the response of a bistable system to a weak coherent signal superimposed to a stochastic input [1]. One commonly expects that an increase of the noise level leads to a deterioration of the performance. In such systems, however, noise can induce synchronized jumps between the two stable states, thus enhancing the output signal. The response shows a resonance–like behavior versus the input noise level, i.e. an optimum value of the latter yielding a maximum in the output amplitude.

The observation of SR in vertical cavity surface emitting lasers (VCSELs) has been previously reported both for periodic [2,3] and aperiodic input signals [4,5]. In particular, the experimental setup allows one to change over a large range the input parameters, such as the modulation frequency and amplitude, noise intensity, and bandwidth. These features, together with the high stability of the system, allow an unprecedented comparison with theoretical works; for example, the study of SR as a function of modulation frequency and a complete characterization of the SR by means of probability distributions have been reported.

In this work we present an experimental study of phase synchronization regimes in SR. We analyze temporal dynamics and statistical properties of the phase difference variable, as reconstructed from the experimental time series. We show that higher degrees of synchronization are achieved increasing the modulation amplitude, in agreement with previous theoretical studies [6–8].

2. Stochastic Resonance in VCSEL

We employ a VCSEL lasing at 850 nm, thermally stabilized (better than 1 mK), and with a carefully controlled pump current. The overall stability allows for long time measurements, even in the presence of critical behaviors. Two linear polarization directions are defined in the laser emission and can be selected using a polarizer and a half–wave plate. The laser intensity is monitored by an avalanche detector and the signal is recorded by a digital scope. An optical isolator prevents from optical feedback effects. The signals from a 10-MHz-bandwidth white-noise generator and a sinusoidal oscillator are summed and coupled into the laser by means of a bias tee.

The emission of VCSELs is characterized by strong polarization fluctuations, due to their almost-cylindrical symmetry. Small anisotropies lead to a selection of two perpendicular polarization directions (along the $[1\bar{1}0]$ and [110] axes of the semiconductor crystal), although laser light can be emitted on both polarizations, depending on the laser structure and on the pump current. As a consequence, the polarization noise is often much larger than the intensity noise. Furthermore, VCSELs show a complicated spatial emission in the transverse plane due to the large Fresnel number of the laser cavity. A phenomenon often observed, and crucial to the present work, is the switching of transverse modes from one polarization direction to the other, which occurs for some particular values of the pump current. In such regions, the laser emission is characterized by noise-driven polarization flips, marking the transition between two different transverse mode configurations with corresponding different polarized intensities.

When a modulation of the current is applied we observe a slightly modulated intensity superimposed to the random jumps between the two above-mentioned levels. Increasing the amount of noise (summed to the pump current), the jumps between the two states tend to synchronize with the applied modulation. In this case, the output modulation is much stronger than without the input noise. Increasing the noise further, the synchronization is lost due to frequent jumps, yielding quite a noisy output. This is the typical signature of stochastic resonance. We emphasize that we do not observe any effect in the total laser emission, i.e., without selecting the polarization.

The above phenomenology can be described by means of a Langevin equation

$$\dot{x} = -V'(x) + A\cos\Omega t + F(t) \tag{1}$$

where x is proportional to the output intensity and V is an effective double–well potential. The random process F is assumed to be white and Gaussian and represents the noise added to the pump current modulated sinusoidally with amplitude A and frequency Ω. A calibration of experimental parameters allowed for a quantitative comparison between numerical simulations of (1) and experimental data [3].

3. Phase Dynamics

The instantaneous phase of a real signal $x(t)$ can be defined in a general way employing the concept of the analytic signal. This is accomplished by considering the complex signal $z(t) = x(t) + iy(t)$ where y is defined by the Hilbert transform

$$y(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{x(t')}{t-t'}dt' \,. \tag{2}$$

The phase is than given by the argument of z i.e. $\tan\Phi = y(t)/x(t)$.

For bistable systems alternative and easier definition can be given based on the knowledge of the sequence of switching times t_n only. The simplest is to define Φ as a piecewise–constant function increasing by steps of π at each jump time t_n. Alternatively, Φ can be defined by linear interpolation between subsequent transition events

$$\Phi = n\pi + \frac{t-t_n}{t_{n+1}-t_n}\pi \qquad \text{for} \quad t_n \le t < t_{n+1} \,. \tag{3}$$

Suitably defined instantaneous phases $\Phi_{\rm in}(t) = \Omega t$ for the input and $\Phi_{\rm out}(t)$ for the output, respectively, allow to introduce an instantaneous phase difference

$$\phi(t) = \Phi_{\rm out}(t) - \Phi_{\rm in}(t) \,. \tag{4}$$

From the above definition follows that each transition between the output (resp. input) states increases (resp. decreases) ϕ by π.

Typical time series of ϕ computed from experimental data are shown in Fig. 1. A clear locking regime is observed for a suitable value of the added noise, corresponding to an almost constant phase difference.

Let us define the average frequency

$$\langle\dot{\phi}\rangle = \langle\omega_{\rm out}\rangle - \Omega \,. \tag{5}$$

In Fig. 2 we report the mean output frequency $\langle\omega_{\rm out}\rangle$ as a function of the noise intensity for several values of the input signal amplitude A. Upon increasing A the region of frequency locking, i.e. the range of noise values for which $\langle\omega_{\rm out}\rangle \simeq \Omega$, widens.

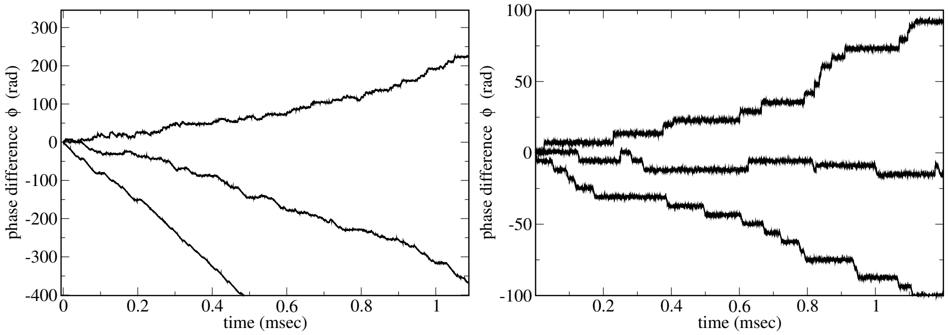

Fig. 1. The istantaneous phase difference $\phi(t) = \Phi - \Omega t$ computed with the Hilbert transform method for input amplitude 100 mV (left) and 250 mV (right). Noise levels are 0.0121, 0.0169, 0.0225 V^2_{RMS} from bottom to top respectively.

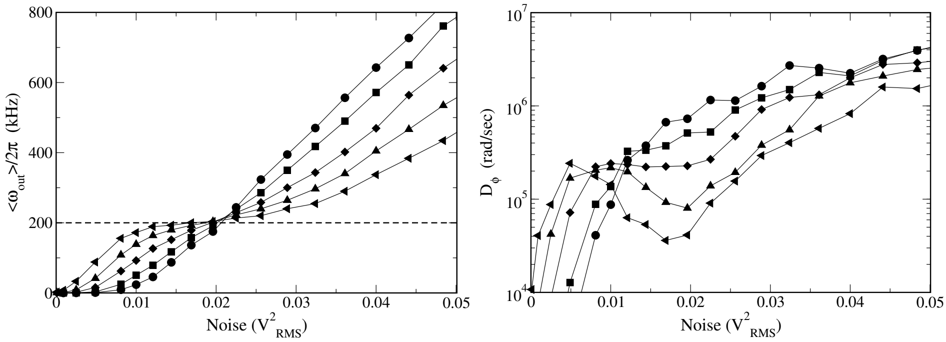

Fig. 2. Experimentally measured average output frequency and diffusion coefficient for input amplitudes 50, 100, 150, 200, 250 mV (circles, squares, diamonds, triangles, left triangles respectively).

The phenomenon of phase locking can be also demonstrated by considering the diffusion coefficient of the phase difference, achieved as the time derivative of the variance

$$D_\phi = \frac{1}{2}\frac{d}{dt}[\langle\phi^2\rangle - \langle\phi\rangle^2]\,. \tag{6}$$

The minimum in the diffusion coefficient indicates strong locking of input and output phases.

The results here presented are in a good qualitative agreement with numerical [6] and analytical [7, 8] studies of the Langevin model (Eq. (1)) and the investigation in electronic circuits [9]. A quantitative comparison of our results with theoretical studies, requiring a careful calibrations of the experimental parameters, is under investigation and will be published elsewhere.

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